IV.

On the Quantitative Determination of an Optical Illusion.

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It is a well-known fact that, under certain conditions,[1] a dotted line appears longer than a point-distance which is objectively equal to it. We do not propose here to deal with the explanations (Hering-Kundt, Aubert, Wundt, *etc.*) proposed for the phenomenon; but rather to describe an attempted numerical determination of the special illusion, with a view to the ascertainment of regularity or irregularity in the course of its absolute or relative sensible discrimination.

Kundt[2] gives particulars of three quantitative series. He and his reagents observed the blackened ends of steel points, 30 mm. long, upon a background of white paper. Five such points were employed: A, B, C and E were kept constant; *D* was moved to and fro, until A*D* was for sensation = *D*E. The data obtained were as

A B C ←*D*→ E

follows. (1) Distance of *D* from the nodal point of the observing eye, 338 mm. Two observers, *R*, *L*, and binocular experiments. Experiments massed = 240. AB = 20.2 mm.; BC = 40.2 mm.; AE = 241.9 mm.; A*D* = 117.64 mm. *Error = 6.62 mm.* (2) Distance, 328 mm. Two observers. *R* and *L* experiments. Experiments massed = 80. AB = 22.6 mm.; BC = 65.5 mm.; AE = 241.9 mm.; A*D* = 116.55 mm. *Error = 8.80 mm.* (3) Distance, 226 mm. One observer. *R*, *L*. Experiments, 40. AB = 25 mm.; BC = 10.5 mm.; AE = 100 mm.; A*D* = 48.52 mm. *Error = 2.96 mm.*[3]

Aubert used four white verticals upon a black background. The distance 1-4 = 100 mm.; 5 was moved to and fro, until 1-4 = 4-5.

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1 2 3 4 5

The experiments, at 2000 mm. distance, gave the error 12 mm.; at 1000 mm., this amounted to 10.5 mm.; at 500 mm., to 11 mm. This latter determination makes it about one-tenth of the whole comparison-distance. Kundt's third series, the conditions of which are similar to those of Aubert's third, had made it as small as one-thirty-fourth. Aubert suggests that Kundt, influenced by knowledge of the illusion, and being his own observer, had unconsciously corrected it.[4]

Before making any remarks upon the results of Kundt and Aubert, we will set forth those obtained in our own experimental series.

New Experiments. We employed, as background, white cards, 125 x 75 mm. in size. The center of each card was the center of the whole line, composed of the dotted line (AB) and the point-distance (BC). BC was kept constant, throughout each of our four card-

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[1] Not under all. If the "dotted line" is a point-distance halved by a single point, it appears smaller than the point-distance. Cf. Mellinghoff in Wundt, *Phys. Psych.*, 4th Ed. II. 142.

[2] Poggendorff's *Ann. d. Phys. a. Ch.*, 1863. Vol. 120 (196). Pp. 118 ff.

[3] These are absolute errors. Thus, in the last case, the equation (A*D*) 48.52 = (*D*E) 51.48 gives the error 2.96. It is with this absolute error that we are dealing. Kundt reckons the error from the objective center of AE; in the case cited = 1.48. He points out, rightly, that for comparative purposes it should be judged not from the objective, but from the subjective (*Augenmass*) center of a point-distance objectively = AE. Cf. Hering, *Beiträge*, 1861, I. 69.

[4] *Physiol. d. Netzhaut*, 266 ff. *Physiol. Optik.*, 630, 1. Cf. Helmholtz, *Physiol. Optik*, 1st Ed. pp. 562-3.

series. It shall, therefore, be termed *C*, in the experimental tables; while AB is termed *V* (variable). The dots in AB (marked in draughting ink with a Keuffel and Esser dotting pen) were 0.5 mm. apart, measured from center to center; and, approximately, 0.3 mm. in diameter. In *Series I.*, BC = 25 mm.; AB varied by half mm. increments from 17 to 27 mm. In *Series II.*, BC = 30 mm.; AB varied between 20 and 34 mm. In *Series III.*, BC = 35 mm.; AB varied between 24 and 38 mm. Finally, in *Series IV.*, BC = 40 mm. AB varied between 29 and 44 mm. Great care was observed that the dots might be equidistant, and of as equal magnitude and clearness as was possible.

The cards were held by the observer at the distance of clear vision (350-400 mm.), and the distances estimated by diffused daylight; the conditions of holding, of illumination, etc., being kept as uniform as possible. For the elimination of constant errors, horizontal judgments were made in equal numbers of *V-C* and of *C-V*; vertical judgments of $\frac{v}{c}$ and of $\frac{c}{v}$. The method employed was a form of the method of minimal changes. We took, *e. g.*, $V < C$, and increased it till it appeared $= C$. This value was noted down. For some few cards further, the subjective equality might persist. So soon as the limit was reached at which *V* appeared $> C$, its value was again noted. Similarly with $V > C$. The mean of these four lengths gave, of course, the estimation value (*R*) of *C* in terms of the dotted line.

Experiments were in no case made upon an observer for more than fifteen minutes consecutively. The manner of judgment differed with different individuals. We were able to distinguish five methods of judging. (*a*) Judgment by reference of the final dots of *C* and *V* to the center and boundary-lines of the card. Curiously enough, *C* appeared to increase (decrease) within limits, as *V* increased (decreased). [1] (*b*) Judgment by superposition of *V* upon *C*, or *vice versa*. (*c*) Judgment in terms of eye-movement. (This is the method presupposed by Wundt, in his discussion of the illusion. For the Hering-Kundt theory it is indifferent whether the eye move or not. Aubert leaves the question of the influence of eye-movement open.) (*d*) Judgment by bisection of the whole line, $V + C$. (*e*) Judgment by memorizing *C*, and comparing a series of *V* with its memory-image. The method (*a*) was employed by one subject only, and only in a few experiments. The method (*e*) must not be too sharply distinguished from (*b*). It is significant that, whatever the method of judgment, the illusion remains. How small a part is played by individual differences in this regard will be shown by our tables. As a strict control of the method of judgment was impossible, it seemed best to allow each observer to follow the line of least resistance. The result was that the principal types,—(*b*), (*c*) and (*d*),—were not infrequently employed in common for the formation of one and the same judgment.

We found that there was manifested in some observers a tendency to regard *C* as the distance from inside edge to inside edge of the limiting dots, while *V* was measured from the outside edge of one to the outside edge of the other. In computing our results, however, we measured both lines from dot-center to dot-center. Each dot is approximately 0.3 mm. wide. *C* was, therefore, seen too short by 0.3 mm.; *V* too long by the same amount. Now the rela-

[1 This suggests an experimental method for the quantitative investigation of apperceptive completion (Wundt, Münsterberg, Külpe, Grützner, *etc.*). Experiments on this subject are at present being carried out in the Cornell laboratory.—E. B. T.]

tive difference-limen for moderate distances is put by Külpe and others at one-fiftieth.[1] A distance of 25 mm. would, therefore, be just noticeably different from one of 24.5; 30 from 29.4; 35 from 34.3; 40 from 39.2. Our values were 24.6, 29.6, 34.6, 39.6. The difference of each of these from its computation-value is, therefore, subliminal, and may be neglected. *C* may stand at 25, 30, 35 and 40 mm. What of *V*? Shall we read, for a *V* of 22, 22.3; for a *V* of 28, 28.3? Plainly, 22 is just noticeably different only from 22.44; 28 from 28.56; *etc.* Again, the difference of the actual from the computation-values is subliminal; the latter may stand. To say that *C* of 25 = *V* of 22, and to say that *C* of 24.6 = *V* of 22.3 are, for psychophysical purposes, the same. It is not as if there were summation of errors, and *C* of 24.3 were compared with *V* of 22.6. In that case, 24.3 would be noticeably different from 25; though 22.6 would not differ from 22. We have mentioned this tendency, although it does not affect our results, because, as can readily be seen, it might under certain conditions of judgment exercise a considerable influence, and demand recognition in the final computation.

All our experiments were made binocularly, and without restriction as to eye-movement. Fairly complete series were obtained from six subjects: Miss Healy (*H.*), Miss Washburn (*W.*), Messrs. Knox (*K.*), Pillsbury (*P.*), Titchener (*T.*) and Watanabe (*Wa.*). The results are subjoined. Under *R* are given the estimation values of the four constants. The *n* column shows the number of records taken, each record including the determination of r_o', r_o'', r_u' and r_u'', from which the average was calculated. Under the rubric *m. v.* is given the total mean variation of each separate *R*, calculated from the whole number of experiments.[2] The mean variation of R_1, R_2, *etc.*, from each *R*, it seems unnecessary to give in a separate column; for the first three constants it lies between one and five tenths mm.; for the last it averages five-tenths,—*W.* having the largest *m. v.*, 1.2 mm., and *Wa.* the smallest, 0.1 mm.

TABLE I.

Reagent *H.* Slight astigmatism[3]. Method *(c)* predominantly follows. Special practice only. Unit = 1 mm.

SERIES.	*R.* C=25	*n.*	*m. v.*	C=30	*n.*	*m. v.*	C=35	*n.*	*m. v.*	C=40	*n.*	*m. v.*
C—V	22.25	3	1.5	26.95	3	2.25	31.41	3	2.58	36.91	3	1.91
V—C	22.00	3	2.0	26.25	3	2.25	30.75	3	2.25	36.91	3	2.08
$\frac{V}{C}$	22.83	3	1.6	27.31	3	2.75	30.83	3	2.00	37.00	3	2.08
$\frac{C}{V}$	22.83	3	1.6	27.33	3	2.00	32.00	3	2.00	37.08	3	2.25
Hor.△[4]	—2.88			—3.40			—3.92			—3.09		
Vert.△	—2.17			—2.68			—3.59			—2.96		

[1] *Grundriss*, pp. 371-2. Wundt, *Vorlesungen*, 2d Ed. p. 162.
[2] This is, of course, not a true m. v. But it is sufficiently accurate for our purpose. Cf. Gruber, Ueber die specifische Helligkeit der Farben, *Phil. Stud.*, IX. p. 435.
[3] Corrected by glasses.
[4] $\Delta = R - r$.

TABLE II.

Reagent *W*. Very slight astigmatism[1]. Methods *(a)* and *(c)*. General and special practice. Unit = 1 mm.

SERIES.	$C=25$	$n.$	$m.v.$	$C=30$	$n.$	$m.v.$	R. $C=35$	$n.$	$m.v.$	$C=40$	$n.$	$m.v.$
$C-V$	23.00	3	1.08	27.66	3	1.45	32.50	3	1.25	38.16	3	1.33
$V-C$	23.33	3	0.91	27.08	3	0.95	31.25	3	1.33	36.83	3	1.16
$\frac{V}{C}$	23.16	3	0.91	27.75	3	1.00	32.13	3	1.08	36.50	3	1.00
$\frac{C}{V}$	22.75	3	0.93	27.75	3	1.00	32.50	3	1.25	37.50	3	1.08
Hor.△	—1.84			—2.63			—3.13			—2.15		
Vert.△	—2.05			—2.25			—2.69			—3.00		

TABLE III.

Reagent *K*. Vision normal. Method *(e)*. General and special practice. Unit = 1 mm.

SERIES.	$C=25$	$n.$	$m.v.$	$C=30$	$n.$	$m.v.$	R. $C=35$	$n.$	$m.v.$	$C=40$	$n.$	$m.v.$
$C-V$	22.50	3	0.62	27.33	3	0.91	32.50	3	0.75	38.16	3	1.33
$V-C$	23.08	3	0.76	27.91	3	0.75	33.08	3	0.83	39.00	3	0.66
$\frac{V}{C}$	23.25	3	1.00	28.25	3	0.75	33.58	3	0.66	37.75	3	0.66
$\frac{C}{V}$	23.33	3	0.79	27.41	3	1.00	33.00	3	0.87	38.08	3	0.91
Hor.△	—2.21			—2.38			—2.21			—1.42		
Vert.△	—1.71			—2.17			—1.71			—2.09		

[1] Not sufficient to call for correction by glasses.

TABLE IV.

Reagent *P*. Vision normal. Method predominantly *(b)*. General and special practice. Unit = 1 mm.

SERIES.	R.											
	$C=25$	*n.*	*m. v.*	$C=30$	*n.*	*m. v.*	$C=35$	*n.*	*m. v.*	$C=40$	*n.*	*m. v.*
$C-V$	23.28	16	0.63	28.25	16	0.64	32.98	16	0.75	37.75	16	0.72
$V-C$	23.82	16	0.69	29.25	16	0.63	34.04	16	0.68	39.00	16	0.59
$\frac{V}{C}$	23.10	10	0.62	27.60	10	0.56	32.34	10	0.48	36.87	10	0.68
$\frac{C}{V}$	23.38	10	0.56	27.72	10	0.60	32.67	10	0.50	36.82	10	0.77
Hor.△	—1.45			—1.25			—1.49			—1.83		
Vert.△	—1.76			—2.34			—2.50			—3.16		

TABLE V.

Reagent *T*. Vision, *L*., normal; *R*., myopic[1]. Method predominantly *(b)*. General and special practice. Unit = 1 mm.

SERIES.	R.											
	$C=25$	*n.*	*m. v.*	$C=30$	*n.*	*m. v.*	$C=35$	*n.*	*m. v.*	$C=40$	*n.*	*m. v.*
$C-V$	20.96	8	0.87	24.03	8	0.96	28.62	8	0.93	33.90	8	0.87
$V-C$	20.62	8	0.78	24.40	8	0.82	29.18	8	0.87	34.56	8	0.81
$\frac{V}{C}$	20.20	5	0.65	23.07	5	0.70	28.05	5	0.85	33.25	5	0.85
$\frac{C}{V}$	20.55	5	1.00	24.40	5	0.70	28.15	5	9.95	34.15	5	0.85
Hor.△	—4.21			—5.79			—6.10			—5 77		
Vert.△	—4.63			—6.27			—6.90			—6.30		

[1]See below, *Remarks* (5).

TABLE VI.

Reagent *Wa*. Vision slightly myopic. Method mixed. General and special practice. Unit = 1 mm.

SERIES.	*C*=25	*n.*	*m. v.*	*C*=30	*n.*	*m. v.*	*R.* *C*=35	*n.*	*m. v.*	*C*=40	*n.*	*m. v.*
C—V	23.41	3	0.58	28.75	3	0.66	32.91	3	0.58	39.55	3	0.58
V—C	23.75	3	0.41	29.25	3	1.58	33.08	3	0.58	37.91	3	1.08
$\frac{V}{C}$	22.41	3	0.66	26.00	3	0.75	33.33	3	0.58	35.91	3	0.25
$\frac{C}{V}$	24.33	3	0.83	30.16	3	0.75	34.58	3	0.50	39.25	3	0.50
Hor.△	—1.42			—1.00			—2.01			—1.27		
Vert.△	—1.63			—1.92			—1.05			—2.37		

Remarks. We notice (1) that the illusion holds for every observer, for both vertical and horizontal judgments, and whatever the method of judgment.

(2) Since our judgment of vertical distances is in general less accurate than our judgment of horizontal (Chodin, Volkmann), we should expect to find a higher value of △ in the former case than in the latter. The twenty-four comparisons of our tables give fourteen confirmations, ten refutations of this expectation. Two of the contrary cases, however, may be regarded as neutral (2.83, 2.17; 2.63, 2.25): so that we have fourteen *r*, eight *w*, two =. Of these eight, four belong to Table I.; two to Table III.; one each to Tables II. and VI. The results of Table I. were taken (*a*) from a previously unpractised subject, who (*b*) began with horizontal judgments. These latter, therefore, may well be regarded as belonging to a stage of less complete practice than the vertical judgments. We have, then, in conclusion, fourteen *r*, four *w*, two =. When we consider the fewness of our experiments, we cannot but think that this is as strong a confirmation of our expectation as could have been hoped for.

(3) Binocular bisection of horizontal distances is not subject to any constant error; binocular bisection of verticals is subject to the constant error of over-estimation of the upper part of the field of vision.[1] We should, therefore, expect to find the *m. v.* of our vertical △'s greater than that of our horizontal. The results, if Table I. is omitted, are: *r* 6, = 7 (.99, .92; .79, .76; .66, .59; .63, .58; .82, .82; .90, .85; .58, .54), *w* 7. [Table I. gives *r* 2, = 1 (1.7, 1.6), *w* 1.] This is curious. We are unable to offer any explanation of the result.

(4) Is there any constancy of the relative or absolute sensible discrimination? Of the latter, obviously not. The values of $\frac{\triangle}{r}$ are:[2]

[1] Wundt, *Grundzüge*, 4th Ed., II. 139, 140.
[2] *Op. cit.*, I. 343.

I. *Hor.*: $\frac{1}{8}$-$\frac{1}{9}$, just over $\frac{1}{9}$, just over $\frac{1}{9}$, just over $\frac{1}{13}$. *Vert.*: $\frac{1}{11}$-$\frac{1}{12}$, $\frac{1}{11}$-$\frac{1}{12}$, $\frac{1}{9}$-$\frac{1}{10}$, $\frac{1}{13}$-$\frac{1}{14}$.

II. *Hor.*: $\frac{1}{13}$-$\frac{1}{14}$, $\frac{1}{11}$-$\frac{1}{12}$, $\frac{1}{11}$-$\frac{1}{12}$, just over $\frac{1}{16}$. *Ver.*: $\frac{1}{12}$, $\frac{1}{13}$-$\frac{1}{14}$, $\frac{1}{13}$, $\frac{1}{13}$.

III. *Hor.*: $\frac{1}{11}$-$\frac{1}{12}$, $\frac{1}{12}$-$\frac{1}{13}$, $\frac{1}{15}$-$\frac{1}{16}$, $\frac{1}{28}$. *Vert.*: $\frac{1}{14}$-$\frac{1}{15}$, just over $\frac{1}{14}$, just over $\frac{1}{21}$, $\frac{1}{19}$.

IV. *Hor.*: $\frac{1}{17}$, $\frac{1}{24}$, $\frac{1}{23}$-$\frac{1}{24}$, $\frac{1}{22}$. *Vert.*: $\frac{1}{14}$, $\frac{1}{12}$-$\frac{1}{13}$, $\frac{1}{14}$, $\frac{1}{12}$-$\frac{1}{13}$.

V. *Hor.*: $\frac{1}{6}$, $\frac{1}{5}$-$\frac{1}{6}$, $\frac{1}{5}$-$\frac{1}{6}$, just over $\frac{1}{7}$. *Vert.*: $\frac{1}{5}$-$\frac{1}{6}$, $\frac{1}{4}$-$\frac{1}{5}$, $\frac{1}{5}$-$\frac{1}{6}$, $\frac{1}{6}$-$\frac{1}{7}$.

VI. *Hor.*: $\frac{1}{17}$-$\frac{1}{18}$, $\frac{1}{30}$, $\frac{1}{14}$-$\frac{1}{15}$, $\frac{1}{31}$-$\frac{1}{32}$. *Vert.*: $\frac{1}{15}$-$\frac{1}{16}$, $\frac{1}{15}$-$\frac{1}{16}$, $\frac{1}{33}$, $\frac{1}{17}$.

This apparent chaos reduces itself to some order on close inspection. To notice are the following facts. (*a*) The values of Table I. are all too large, for the reason already given. Moreover, here the vertical discrimination is (contrary to rule) finer than the horizontal. An explanation of this fact has been also suggested above. (*b*) The irregularities of Table III. may be referred to the fact that the reagent *K.* was the experimenter in all the experimental series of the other tables, and had himself prepared the cards employed in experimentation. (*c*) The smallness of certain fractions in Table IV. may be due to the very large number of experiments made with the reagent *P.* This number was probably *too* large. One of the difficulties in the way of the quantitative determination of an optical illusion is just this, — that the observer must be practised enough to be a good discriminator, but not so practised that he minimizes the illusion by correction. (*d*) Table V. we shall deal with separately, later. (*e*) The $C = 40$ mm. gives throughout a smaller fraction than we should expect. The reason for this does not, however, lie in any of the conditions of the illusion under investigation, but in the nature of our apparatus. Our cards were 125 mm. long. The 40 mm. series of lines varied between 69 and 84 mm. The average length of the line, the dotted part of which was judged equal to 40 mm., was 77 mm. Plainly, the terminal dots of this are dangerously near the card-edges: the distance on either side amounts only to 24 mm. When the cards were being prepared, the question was raised as to whether the 40 mm. series could safely be introduced, or whether it would not be better to use a larger sized card. We decided to test the matter; and our results show decisively the determining influence of the too-near limiting line. (*f*) Table VI. is puzzling. Want of practice will not explain the heterogeneousness of the $\frac{\triangle}{r}$ values; for Table I. shows a clear uniformity, on the basis of very much less practice. We are inclined to suspect a more or less sporadic tendency on the part of the reagent to correct the illusion. Such a tendency is hard to guard against; and conscious avoidance of it may easily lead to exaggeration of the illusion.

Premising that the values of Table V. are explicable, and taking into account all the facts above enumerated, we would conclude from the experimental results,—though quite tentatively, and subject to correction by further investigation,— that *within the limits $C = 25$-40 mm. there is a constancy of $\frac{\triangle}{r}$, at about $\frac{1}{13}$ for horizontal estimation, and at a somewhat higher value for vertical.* For the latter determination, see, especially, Tables IV. and V.

(5) How are the results of Table V. to be explained? We notice (*a*) that it is a typically good table, as regards the illustration of the uniformities which we have discussed. $\frac{\triangle}{r}$ is constant; the vertical $\triangle$'s are greater than the horizontal; discrimination is affected by the "boundary-error" in the 40 mm. judgments; the

m. v. shows 1 *r*, 1 *w*, and 2 = cases. (*b*) But the numerical values of $\triangle$ and $\frac{\triangle}{r}$ are far too high.

The reagent *T.* proposed and planned the investigation. He had more familiarity than any of the other five subjects with the phenomena of optical illusion. He, therefore, approached the experimentation with a definite expectation of what would, in general, result from it; and he endeavored to give the illusion full play, being aware of the tendency towards correction which is apt to arise in the course of a sustained consideration of similar phenomena. It may be that this expectation and endeavor combined to bring about an exaggeration of the illusion; just as the knowledge and practice of the reagent *P.* combined to decrease it.

But *T.'s* vision is also not normal; *L.* being emmetropic, *R.* myopic.[1] All the judgments of Table V. were made with the naked eye; *i. e.*, monocularly, by *L.* Could this fact have influenced the estimation? In the cases of horizontal distances, Kundt was able in Series I. to mass his *L.*, *R.* and binocular results, (i) for a far-sighted subject, (ii) for his own unequal (*normal-kurzsichtig*) vision.[2] In Series II. he massed two sets of judgments, one made principally with *L.* the other with R.[3] Series III. gave a subliminal difference for his own unequal *R.* and *L.*[4] (*L.* = 48.26 mm.; *R.* = 48.78. AE = 100.) In determining the subjective middle of AE = 100, as point-distance, he found that the *L.*-judgment gave 50.33 for the left half, the *R.*-judgment 49.845 for the same half.[5] He observes, after calculating the *R.* and *L.* errors of Series III. on this basis, that "sich die Verschiedenheit meiner Augen wieder sehr bemerklich macht;"[6] but it is plain that the "very noticeable difference" is quite inadequate to explain the figures of our Table V. We must, therefore, lay the main stress upon the explanation which we first suggested.

Supplementary Remarks. (1) We have already noticed Mellinghoff's statement, that a point-distance halved by a dot appears smaller than a simple point-distance, which is objectively equal to it. It would be interesting to investigate systematically the quantitative relation of the illusion under discussion to the number of dots. We prepared a series of cards, upon which were two point-distances, *AB* and *BC*, of 30 mm. length. *BC* was varied in ten ways: left open, halved by one dot, divided up at equal intervals by 2, 3 . . . 9 dots. Some ten judgments gave the following constant result:—

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[1] Formula for reading glasses: *L.*, convex, 0.25 + *R.*, concave, 1.25 dioptr. For long-distance glasses: *L*, concave, 0.00 + *R.*, 2.75 dioptr.

[2] *L. c.*, p. 132.

[3] P. 133.

[4] P. 134.

[5] P. 135.

[6] P. 136, note.

REAGENT:

Dots in BC	H.	K.	P.	T.	Wa.
0	=	=	=	=	=
1	=	—	—	—	—
2	=	+	+	+	=
3	+		Illusion constantly increases	↓	Small +
4				Maximum +	"
5	↓	↓		↑	"
6	Maximum +	Maximum +			"
7	↑	↑			"
8					"
9	+	+			"
				+	

Obviously, here is a problem, the investigation of which promises to bring to light new uniformities.

(2) *Practice.* The constancy and extent of the influence of practice on these experiments cannot be better shown than by the presentation of a series of observations from the results of the observer *P.* The series is *V-C; C* = 25.

Oct. 12, 1893.	*R.* = 22.87	m. v. = 0.62
" 18,	23.00	0.50
" 20,	23.12	1.12
" 23,	23.25	0.62
" 25,	24.00	0.75
" 26,	23.87	0.87
" 28,	24.12	0.75
Nov. 2,	23.37	1.37
" 3,	24.50	0.75
" 5,	25.12	1.12
" 9,	24.87	0.43
" 10,	24.37	0.43
" 11,	24.87	0.62
Dec. 6,	23.75	0.50
" 8,	24.75	0.25
" 10,	24.00	0.50

etc.

Practice has already been discussed above: *Remarks* (4).

(3) Can we compare our $\frac{\Delta}{r} = \frac{1}{13}$ with the values obtained by Kundt and Aubert? Aubert's $\frac{1}{10}$ is tempting; but he was dealing with vertical lines, not with points. And his filled space contained only two verticals. We have seen, under (1), that the character of the filling is essential in the illusion. Kundt employed points, it is true, but his distribution of them was curiously arbitrary. And his experiments are, on other grounds, hardly comparable with our own; so that the divergence of the two sets of results need not trouble us.

[Further experiments upon this special illusion are in progress in the laboratory.—E. B. T.]